2. Surface Wettability *[partial solutions]*

In this exercise, we explore the importance of wettability in the behaviour of surfaces. Specifically, we study the problem of capillary rise. This problem is common in manufacturing and arises in various situations, like for dispensing adhesive, ink-jetting, packaging, self-alignment in electronic circuits during soldering or gluing, etc.

2.1. Qualitative Analysis

We consider the system shown in **Figure 4**.

1. Describe the difference between red and blue capillaries.

Red capillaries are *hydrophilic*, blue capillaries are *hydrophobic*.

2. Show the balance of forces at the interfaces:
    a. Between a blue capillary and the liquid

Draw schematics including the gravity force and the surface tension.
You should represent the surface tension in the hydrophobic case.

    b. Between a red capillary and the liquid

Draw schematics including the gravity force and the surface tension.
You should represent the surface tension in the hydrophilic case.

3. Discuss the phenomenon of capillary rise and its opposite effect. What makes a liquid climbing up a capillary or sinking down?

If a surface is hydrophilic and the *surface to volume ratio* is large (i.e. surface tension overcomes hydrostatic pressure), we will witness capillary rise, as it is energetically favourable for the liquid to be in contact with the solid walls (at the expense of a decrease in hydrostatic pressure).

What if the surface is hydrophobic?

2.2. Quantitative Analysis

We consider the case of a single capillary as described in. We assume that the system is in equilibrium and the liquid in the capillary has now reached a height h.

4. In the case of **Figure 5**, is the inner surface of the capillary, hydrophobic or hydrophilic? Justify your answer.

Hydrophilic, why?

5. Using the balance of forces, express the height h that the liquid will reach as a function of the surface tension and the density of the liquid.
Hint. Analyse the balance of forces acting on the liquid meniscus (e.g. pressure and capillary force). To do so, start by finding an expression for F_{cap} based on a dimensional analysis.

$$h = \frac{2\gamma_{LG}\cos(\theta)}{\rho g r}$$

6. Repeat question 5, but this time using Laplace equation as seen in the Lecture. Show that this approach yields an equivalent result as found in the previous question.

Use the Young-Laplace equation assuming that x and y are perpendicular axis defining the liquid surface in Cartesian coordinates.

Then consider a spherical surface with radius R and find ΔP_m.

Finally, remember that the liquid will move until this pressure difference is compensated by the arising hydrostatic pressure $\Delta P = \rho g h$. At this point, the two pressure differences are equal.

You should find that

$$h = \frac{2\gamma_{LG}\cos(\theta)}{\rho g r}$$

7. *Discussion from the viewpoint of designing an element.* Describe for a given surface tension how one can reach the maximum capillary rise. What parameters can be optimized?

Depending on the contact angle θ ($> 90°$ for a hydrophobic surface, $< 90°$ for a hydrophilic one), $\cos(\theta)$ will be positive or negative, leading to a positive or negative value for h. This value indicates whether the meniscus is above or below the surface of the liquid surrounding the tube.

Moreover, reducing the radius of the tube leads to a higher capillary rise (or depression). Not surprisingly, more extreme values (close to $0°$ or $180°$) of θ, indicating more hydrophilic/hydrophobic materials, lead to the same outcome.

Assuming water and air as the liquid and gas, γ_{LG} is a fixed value. Otherwise, a higher surface tension could also lead to a higher capillary raise. However, we have to keep in mind that γ_{LG} and θ are linked by the Young equation $\gamma_{SG} - \gamma_{SL} = \gamma_{LG} \cdot \cos(\theta)$.

3. Surface Analysis *[guided solutions]*

1. Choose an image. Describe it in two and three dimensions. Describe features and identify, if needed, defects and artifacts that may be present in the image.

We will solve this exercise using "Case study 2" image as it contains interesting features. The same procedure could be repeated to the other images.

* *

"Case study 2" image, in 2D (left) and 3D (right).

The images provided represent the surface of a glass sample measured by AFM (Atomic Force Microscope). The glass had been exposed to a laser treatment, followed by a chemical attack in an acid. The time the sample spent in the acid was of 8, 14, 20 and 26 hours for the images 1, 2, 3 and 4 respectively. The exposure to the laser alters the etch rate of the glass (resulting in the lines from the top to the bottom visible on certain images).

The surface exhibits peaks (white) and valleys (black) of different heights.

2. Analyse the surface with the *Roughness* command. Discuss the differences between texture, waviness, and roughness.

As seen in class, a surface can be described by its waviness and its roughness.

The "Calculate roughness parameter" command from Gwyddion allows to draw a line and get the height profile along this line.

The cut-off frequency parameter can be tuned in order to change how features will be interpreted, as roughness of waviness.

Basically, frequencies up to that cut-off will be considered as waviness, and above that cut off will be considered as roughness.

<u>As a conclusion:</u>

- *waviness* is…
- *roughness* is…
- *texture* is the surface shape; it is the addition of waviness and roughness.

*

3. Separate waviness and roughness components by using FFT filtering. We recommend starting with 1D-filtering, to have an understanding of the working principle of a FFT filter, then proceed with 2D-filtering for better precision. Learn how to add/subtract the different contributions. Plot the results in 2D and in 3D.

As we know that waviness is a ______ frequency component, and ______________ is a high frequency component, we can compute the Fourier transform (FFT: Fast Fourier Transform) of the image to get into the frequency domain (Data Process > Correct data > FFT filtering).

<u>Let's start with 1D FFT filtering:</u>

1D FFT computes the Fourier transform of the image in ____ __________ only (either _________ or _________) and displays it.

Sliding the mouse on the Fourier spectrum graph allows to select a frequency window (in purple) that will be **discarded/filtered out**, hence displaying the image without these frequencies.

*

By cutting off the high frequencies, only the low frequencies, i.e. the ______________, will remain on the image.

* *

Low-pass filtered image (high frequencies cut out).

By cutting off the low frequencies, only the high frequencies, i.e. the ______________, will remain on the image.

* *

High-pass filtered image (low frequencies cut out).

<u>2D FFT filtering:</u>

The underlying principle in the same as 1D FFT, except here the horizontal and vertical Fourier transform are made at the same time.

The orange area corresponds to the frequencies **kept**, and the rest is cut out.

* * * *

Applying a Fourier mask in only one direction allows for filtering _____________________, e.g. cutting the vertical of the Fourier transform highlights the _____________________, whereas cutting out the horizontal of the Fourier transform highlights _____________________.

The 2D Fourier transform is actually a 2D image, symmetric, with ______ frequencies in the centre and ______ frequencies towards the edges. We can perform similar operations as 1D filtering, but in several directions.

* *

Low-pass filtered image (high frequencies cut out).

* *

Low-pass filtered image (high frequencies cut out).

4. Compare the plots obtained in Question 3 with the profiles investigated in Question 2. Are the profiles of waviness and roughness similar? Why?

We can indeed measure the waviness and roughness by taking a line profile on the Fourier filtered images. A line profile along the low-pass filtered image will give us the _______________, and a line profile along the high-pass filtered image will give us the _______________.

* *

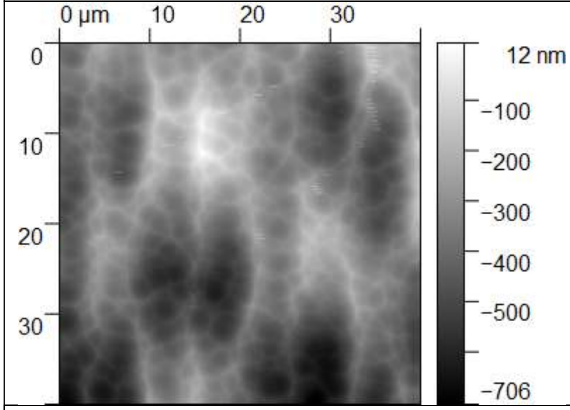
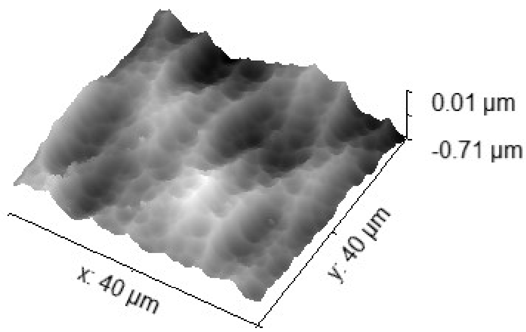
Left: arbitrary line profile taken in the low-pass filtered image (i.e. _______________).
Right: arbitrary line profile taken in the high-pass filtered image (i.e. _______________).

5. Repeat questions 1 to 4 with different images to study the effect of the etching parameters on the surface texture. Do you notice particular surface textures with preferred orientation?

An interesting feature with sample 2 is that it exhibits a high peak at a specific frequency. This means that there is a _________________________________, in that case every __ μm (the peak in the Fourier transform is located at $k = \_\_ \ \mu m^{-1}$).
This could be due to _________________________________, as the spot used in this kind of experiment is typically around $\sim 1\ \mu m$ diameter.

*

We can highlight where this period appears in the image by filtering it out and comparing the two images:

Original image	Image with filtered peak
	*
	*

It is strongly advised to play with the other images as this will give you the chance to develop a more intuitive grasp on the characterization of a surface, as well as to understand the influence of etching time on surface topology.

4. Bennett and Porteus Model for Reflectivity *[partial solutions]*

During and after manufacturing, polishing requirements must be carefully considered depending on the function that a surface shall fulfil. *Optical surfaces* typically have stringent requirements in terms of surface roughness, at the price of a certain cost. In general, the lower the roughness, the higher the cost.

To investigate this problem, we analyze the Bennett and Porteus model seen in class:

$$R_s(R_q) = R_0\left(1 - e^{-\left[4\pi\left(\frac{R_q}{\lambda}\right)\cos(\theta_i)\right]^2}\right)$$

where R_s is the total scattered intensity, R_0 the intrinsic reflectance of the surface (i.e., that of a perfectly smooth surface of the same material), R_q the RMS roughness of the considered surface, λ the incident wavelength and θ_i the angle of incidence of light (defined with respect to the normal to the surface, as shown in **Figure 6**).

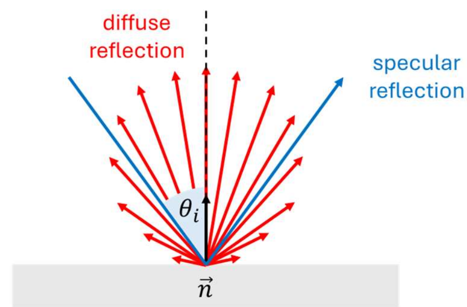


Figure 27. *Representation of the light interaction with a rough surface.*

4.1. Mirror Surface

1. We would like to produce a ‘good’ mirror surface, that is, as least as possible visible. How should we optimize the ratio between the scattered and intrinsic reflectance?

Minimize R_s/R_0.

2. Using your favourite plotting program (Matlab or others), plot a normalized, dimension-free representation of the scattered intensity for various angles (i.e., the ratio R_q/λ versus R_s/R_0).

You need to plot the curves as explained to have a graph!

3. Discuss the influence of the angle of incidence of a light beam on the scattered intensity.

The angle of incidence has a strong impact on the reflective scattered intensity versus intrinsic reflective intensity. Note how rapid is the dependence of the scattered intensity with wavelength as the angle approaches quasi-grazing angle ($\theta_i = \pi/2$).

4. Let us assume now that the angle of incidence is $\pi/4$. What should be the roughness to achieve a maximum of 10% scattered intensity for a wavelength of (a) $10\ \mu\text{m}$? (b) $0.5\ \mu\text{m}$? (c) $0.2\ \mu\text{m}$? *Note. This is the typical wavelength of CO_2 lasers, widely used in laser cutting applications.*

Let us use the graph from the Question 2: (a) $300\ \text{nm}$ of RMS surface roughness, (b) $15\ \text{nm}$ of RMS surface roughness, (c) $6\ \text{nm}$ of RMS surface roughness.

5. Consider the tables providing the arithmetical roughness R_a for various types of processes (see Lecture 2, slides 26-27). Which process(es) could be considered to achieve the scenarios discussed in (a) Question 4(a)? (b) Question 4(b)? (c) Question 4(c)?

We computed the required values for R_q, the RMS roughness. We know that this figure of merit gives more importance to the large peaks and valleys compared to R_a. At a first approximation, we can hence assume that a machining process satisfying $R_a = R_{q,a/b/c}$ will satisfy the R_q requirements. Some careful milling might be sufficient for case (a). However, only diamond turning or fine polishing methods can fulfil the roughness requirements for cases (b) and (c).

6. What angle could we choose to make the surface as reflecting as possible and to minimize the scattered intensity?

We see that for $R_s/R_0 = 10\%$, if the angle approaches the *grazing angle* $\theta_i = \pi/2$, the scattered intensity can be reduced to a percent, with still the same roughness.

4.2. Measurement of the Surface Roughness

7. Now, we would like to perform a surface roughness characterization by measuring the reflectivity properties of a given surface. For this purpose:
   a. What figure of merit should you measure?

R_q may be obtained by measuring the total diffuse reflectance (i.e., total scattered intensity) R_s as this is the signal due to the interaction of the incident light with the surface roughness.

   b. What angle θ_i should the incident light have?

Now, we want to maximize the R_s/R_0 ratio and hence shine light normal to the surface ($\theta_i = 0°$).

   c. Would you prefer an incident light with longer or shorter wavelengths?

Longer wavelengths are better, and this can be easily understood from a physical point of view.

Consider a nominally plane surface made up of many small facets randomly oriented in various directions. If the dimensions of the facets are large compared with the wavelength of light, the reflectance of a surface in a given direction is determined entirely by geometrical optics and is a function only of the inclinations of the facets.

As the wavelength becomes longer, diffraction effects become important, and the reflectance is a function of both the inclination and the size of the facets. As the wavelength still increases so that the dimensions of the facets become very small by comparison, the reflectance of the surface will be determined almost entirely by diffraction effects. In other words, the surface roughness will then be the only important parameter.

8. Express R, the specular reflectance of the rough surface at normal incidence.

Write the initial formula at normal incidence: $R_s = ?$
Now, observe that R_s is the difference between the intrinsic reflectance of the surface and a similar quantity corresponding to the specular reflectance due to the roughness.
This makes sense as if $R_q \to 0$, $R_s \to 0$. By identification, you conclude that $R_s =$ ___ − ___, so R is

$$R = R_0 e^{-\left[4\pi\left(\frac{R_q}{\lambda}\right)\right]^2}$$

9. Starting from the expression for R found in the previous question and assuming that $R/R_0 \to 1$, derive an expression for the RMS roughness of the surface as a function of R and R_0.

Use the first term of the Taylor series of $\ln(x)$ when $x \to 1$ (as we assumed $R/R_0 \to 1$):

$$T_{x\to 1}(\ln(x)) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \cdots$$

Using this approximation into our equation, you should find that

$$R_q = \frac{\lambda}{4\pi}\sqrt{\frac{R_0 - R}{R_0}}$$

10. Finally, give an expression for the total diffuse reflectance and interpret the result.

$$R_s = R_0 \left(\frac{4\pi R_q}{\lambda}\right)^2$$

Interestingly, if the ratio R_s/R_0 is measured at a fixed wavelength, the surface roughness R_q is directly proportional to $\sqrt{R_s/R_0}$ (hence the curve obtained in Question 2!).